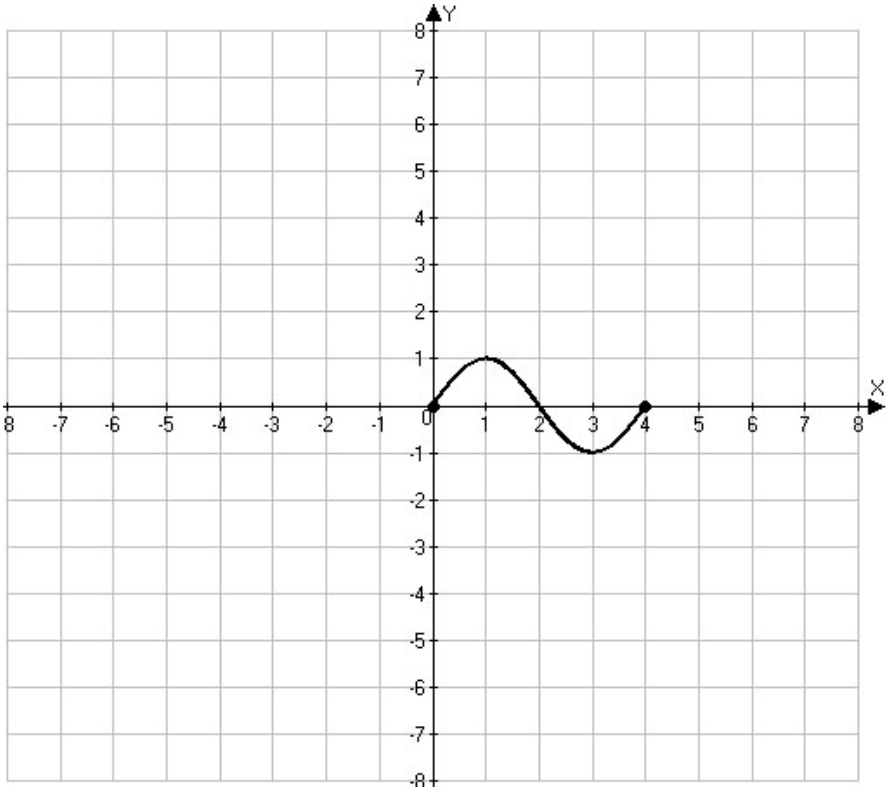
Math 5 – Trigonometry – fall '08 – Chapter 3 Test Name_____
Show all work for credit. With the exceptions of #4 and #7 write all responses on separate paper.

1. Consider the line that intersects $f(x) = 2(x-1)^2 - 2$ where $x = -1$ and where $x = 4$.
   a. Find the slope of the line.
   b. Find an equation for the line.
   c. Graph the line and the parabola $y = f(x)$ together showing the points of intersection, and the parabola's vertex and intercepts.

2. The domain of each function below is all real numbers.
   Express the range of each function using interval notation.
   a. $f(x) = 3x^2 - 12x + 13$
   b. $g(x) = \dfrac{1}{3(x-2)^2 + 1}$

3. Consider the square root function, $f(x) = 2\sqrt{x+1}$
   a. Write the domain and range of this function using interval notation.
   b. Make a table of values for the function and sketch its graph showing the intercepts and at least two other points.
   c. What transformations (shift(s) and/or stretch/shrink) would be required to transform this function to $y = \sqrt{x}$?

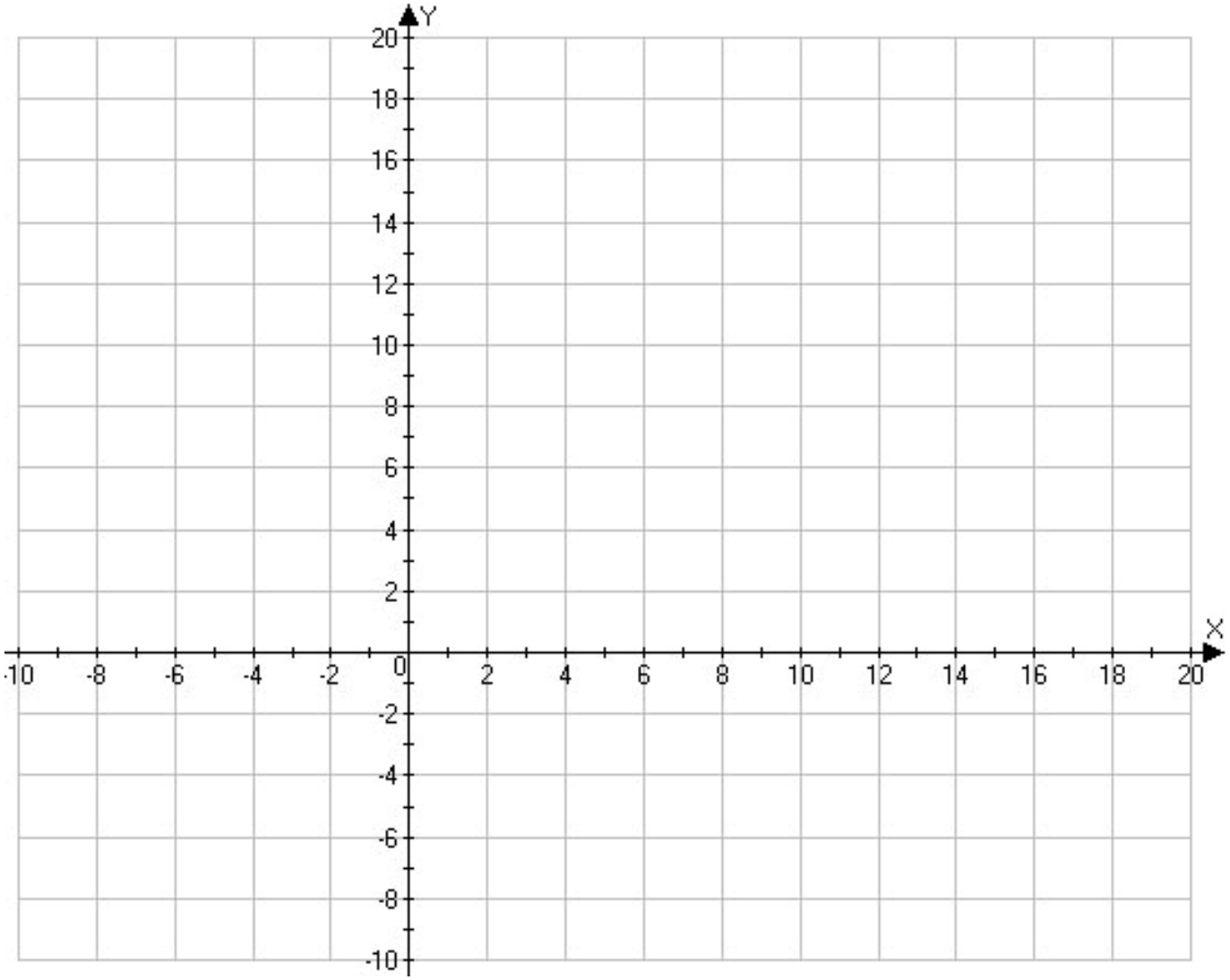
4. Given the graph of $y = f(x)$ shown at right, graph and label the following transformations in the space provided.
   a. $y_1 = f(x+4)$
   b. $y_2 = 3 + f\left(\dfrac{x}{2}\right)$
   c. $y_3 = f(-x) - 4$



5. The reciprocal of one number is 2 more than the reciprocal of another number. Express the product of the numbers as a function of a single variable.

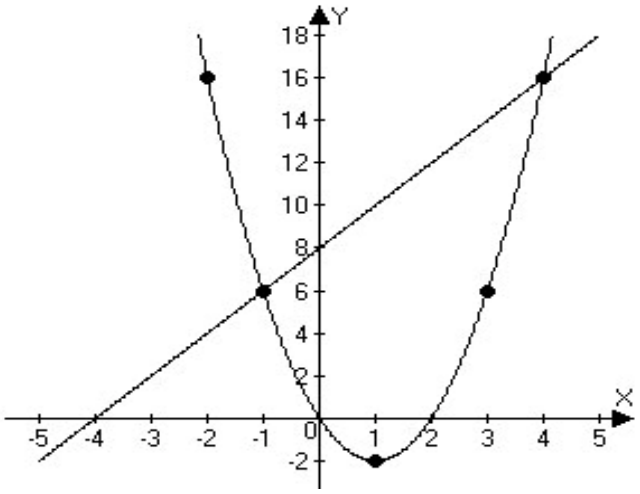
6. Suppose $f(x)=\frac{1}{x-1}$ and $g(x)=\frac{1}{x-2}$. Find the domain of $(f\circ g)(x)$

7. Consider the function $f(x)=\sqrt[3]{x+8}-3$
   a. Find an inverse function formula for *f*. *Hint*: This is a cubic polynomial formula.
   b. Tabulate (x, y) pairs for $y=f(x)$ for $x=-8$, $x=-7$, $x=0$ and $x=19$.
   c. Use this table to sketch graphs for $f^{-1}(x)$ and $f(x)$ together showing the symmetry through the line $y=x$.



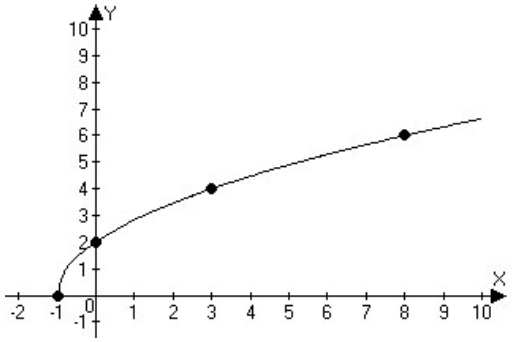
Math 5 – Trigonometry – fall '08 – Chapter 3 Test Solutions

1. Consider the line that intersects $f(x) = 2(x-1)^2 - 2$ where $x = -1$ and where $x = 4$.
   a. Find the slope of the line.
      SOLN: $\dfrac{f(4)-f(-1)}{4-(-1)} = \dfrac{2(4-1)^2-2-\left(2(-1-1)^2-2\right)}{5} = \dfrac{(18-2)-(8-2)}{5} = 2$
   b. Find an equation for the line.
      SOLN: Using the point slope equation and the point (4, 16) we get
      $y-16 = 2(x-4) \Leftrightarrow y = 2x+8$ a
   c. Graph the line and the parabola $y = f(x)$ together showing the points of intersection, and the parabola's vertex and intercepts



2. The domain of each function below is all real numbers.
   Express the range of each function using interval notation.
   a. $f(x) = 3x^2 - 12x + 13$
      SOLN: $f(x) = 3x^2 - 12x + 13 = 3(x-2)^2 + 1$ has a range of $[1, \infty)$
   b. $g(x) = \dfrac{1}{3(x-2)^2+1}$ is the reciprocal of f so its rang is $(0,1]$.
3. Consider the square root function, $f(x) = 2\sqrt{x+1}$
   a. Write the domain and range of this function using interval notation.
      SOLN: Domain = $\{x \mid x+1 \geq 0\} = [-1, \infty)$ and range = $[0, \infty)$
   b. Make a table of values for the function and sketch its graph showing the intercepts and at least two other points.

x	y
-1	0
0	2
3	4
8	6



   c. What transformations (shift(s) and/or stretch/shrink) would be required to transform this function to $y = \sqrt{x}$?
      SOLN: Shrink vertically by a factor 2 and shift 1 to the right.
      Alternatively, stretch horizontally by a factor 4 and shift 4 to the right.

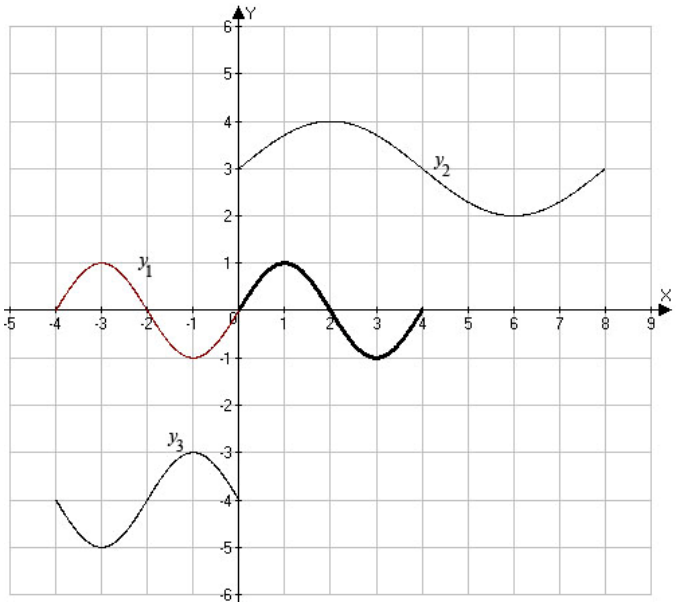
4. Given the graph of $y = f(x)$ shown at right, graph and label the following transformations in the space provided.

   a. $y_1 = f(x+4)$

   b. $y_2 = 3 + f\left(\frac{x}{2}\right)$

   c. $y_3 = f(-x) - 4$

   Note the last one is visually equivalent to shifting 4 left and 4 down, though that doesn't flip it, which the reflection



5. The reciprocal of one number is 2 more than the reciprocal of another number. Express the product of the numbers as a function of a single variable.

   SOLN: Let the numbers be x and y. Then $\frac{1}{x} = \frac{1}{y} + 2 = \frac{1+2y}{y} \Rightarrow x = \frac{y}{2y+1} \Rightarrow xy = f(y) = \frac{y^2}{2y+1}$

6. Suppose $f(x) = \frac{1}{x-1}$ and $g(x) = \frac{1}{x-2}$. Find the domain of $(f \circ g)(x)$

   SOLN: $(f \circ g)(x) = f\left(\frac{1}{x-2}\right) = \frac{1}{\frac{1}{x-2} - 1} = \frac{x-2}{3-x}$ has domain $(-\infty, 2) \cup (2, 3) \cup (3, \infty)$.

7. Consider the function $f(x) = \sqrt[3]{x+8} - 3$

   a. Find an inverse function formula for f. *Hint*: This is a cubic polynomial formula.

   SOLN: $y = \sqrt[3]{x+8} - 3 \Leftrightarrow \sqrt[3]{x+8} = y+3 \Leftrightarrow x+8 = (y+3)^3 \Leftrightarrow x = (y+3)^3 - 8$ so

   $f^{-1}(x) = (x+3)^3 - 8$

   b. Tabulate (x, y) pairs for $y = f(x)$ for $x = -8$, $x = -7$, $x = 0$ and $x = 19$.

x	-8	-7	0	19
$\sqrt[3]{x+8} - 3$	-3	-2	-1	0

   c. Use this table to sketch graphs for $f^{-1}(x)$ and $f(x)$ together showing the symmetry through the line $y = x$.